

MECHANICS 3 (A) TEST PAPER 1 : ANSWERS AND MARK SCHEME

1. $L \sin \theta = mv^2/r = 0.5 \times 36 \div 2 = 9$ $L \cos \theta = 0.5g = 4.9$ M1 A1 M1 A1
 $\tan \theta = 9 \div 4.9 = 1.84$ $\theta = 61.4^\circ \approx 61^\circ$ M1 A1 A1 7
2. (a) $T = \frac{\lambda(0.1)}{0.2} = \frac{\lambda}{2}$ Resolve vertically : $2T \cos 30^\circ = 0.7g$ M1 A1 M1 A1
 $T = 3.96$ $\lambda = 2T = 7.92$ M1 A1
- (b) Assumed P is a particle, e.g. a single point at vertex of the Δ B1 7
3. $F = ma : 0.5v \frac{dv}{dx} = -\frac{8}{x^2}$ $\int v dv = -\int \frac{16}{x^2} dx$ B1 M1
 $\frac{v^2}{2} = \frac{16}{x} + c$ $x = 2, v = 4 : 8 = 8 + c$ $c = 0$ $v^2 = \frac{32}{x}$ A1 M1 A1 A1
When $x = 0.5, v^2 = 64$ $|v| = 8 \text{ ms}^{-1}$ M1 A1 8
4. (a) At greatest depth, gravitational P.E. lost = elastic P.E. gained
 $mg(2l) = \lambda l^2/2l$ $\lambda = 4mg$ M1 M1 A1
- (b) Gravitational P.E. lost = elastic P.E. gained + K.E. gained
 $mg \frac{5l}{4} = \frac{4mg}{2l} \cdot \frac{l^2}{16} + \frac{1}{2}mv^2$ $\frac{5gl}{4} - \frac{gl}{8} = \frac{1}{2}v^2$ M1 A1 A1
 $v^2 = \frac{9gl}{4}$ $v = \frac{3}{2}\sqrt{gl} \text{ ms}^{-1}$ M1 A1 A1 9
5. (a) Let mass of cone = M , so mass removed = $\frac{1}{8}M$, remainder = $\frac{7}{8}M$ M1 A1
 $M \frac{h}{4} = \frac{M}{8} \left(\frac{h}{2} + \frac{h}{8} \right) + \frac{7M}{8} \bar{y}$ $\frac{7}{8}\bar{y} = \frac{h}{4} - \frac{5h}{64} = \frac{11h}{64}$ $\bar{y} = \frac{11h}{56}$ M1 A1 A1 M1 A1
- (b) $\tan \alpha = \frac{r}{2} + \left(\frac{h}{2} - \frac{11h}{56} \right) = \frac{28r}{17h}$ $\frac{28r}{17h} = \frac{1}{2}$ $h : r = 56 : 17$ M1 A1 M1 A1 A1 12
6. (a) $mg = \frac{mg}{2l}e$ $e : l = 2 : 1$ M1 A1
- (b) $m\ddot{x} = mg - \frac{mg}{2l}(2l+x)$ $\ddot{x} = -\frac{g}{2l}x$, so S.H.M. M1 A1 M1 A1
- (c) $\omega^2 = \frac{g}{2l}$ Period = $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2l}{g}}$ s M1 A1
- (d) At $E, v = a\omega = \frac{3l}{2}\sqrt{\frac{g}{2l}} = \sqrt{\frac{9gl}{8}}$ ms^{-1} M1 A1
- (e) $x = \frac{3l}{2} \cos \omega t$ $x = -\frac{3l}{4}$ when $-\frac{3l}{4} = \frac{3l}{2} \cos \omega t$ M1 A1 A1
 $\cos \omega t = -\frac{1}{2}$ $\omega t = \frac{2\pi}{3}$ $t = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{\frac{2l}{g}}$ s M1 A1 15
7. (a) K.E. lost = P.E. gained : $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgl - mgl \cos \theta$ M1 A1
 $v^2 = u^2 - 2gl + 2gl \cos \theta$ A1
 $T - mg \cos \theta = \frac{mv^2}{l}$ $T = \frac{m}{l}(u^2 - 2gl + 2gl \cos \theta) + mg \cos \theta$ M1 A1 M1 A1
 $T = 0$ when $\theta = 120^\circ : \frac{u^2}{l} - 2g - g - \frac{g}{2} = 0$ $u^2 = \frac{7gl}{2}$ M1 A1 A1
- (b) Now moves as projectile with initial speed v , where $v^2 = u^2 - 3gl$ M1
so $v = \sqrt{\frac{gl}{2}}$, and angle of projection 60° A1 B1
At highest point above $O, h = \frac{l}{2} + \frac{v^2 \sin^2 60}{2g} = \frac{l}{2} + \frac{gl}{4g} \cdot \frac{3}{4} = \frac{11l}{16}$ m M1 A1 M1 A1 17